The paper presents a comparative analysis of five models describing the changes in BOD\textsubscript{5} based on results of the experimental research. The model of Thomas (1950), Navone (1960), Fujimoto (1964), Adrian and Sanders (1992–1993) as well as Young and Clark (1965) used by Adrian and Sanders (1998) were studied. The values of BOD\textsubscript{5} were measured for 5 days with the measurement step of 3 h in every sample out of 92. Therefore, each course of the variation was described by 40 measurement points. To define the influence of the processes of nitrification on the level of BOD\textsubscript{5}, each sample was examined with addition of an inhibitor of nitrification and without it. It was found that reaction constant did not depend on the final value of BOD in the models of the first order (Thomas, Navone and Fujimoto) contrary to the models of the second order (Young and Clark), and especially the half order (Adrian and Sanders). It is the easiest to define the lag phase of the time dependence BOD by the model of Thomas, which clearly depicts the border between the growth sequence of BOD and the initial fluctuations of the growth. The influence of the nitrification inhibitor on the course of the BOD was not noticed in all samples. It suggests that the process of nitrification occurred in a part of the samples almost from the first day and more often from the second or third day and in others this process did not occur in the whole measurement period (5 days) at all.

1. INTRODUCTION

The biochemical oxygen demand (BOD) is an indicator used to evaluate the degree of the contamination of water and sewages with the organic substances. The total aerobic decomposition of organic compounds occurs for about 100 days. During the first 5 days 60–70\% of the organic substances decompose [7], whereas during the period of about 20 days decompose carbohydrates and then other compounds including the organic nitric compounds [11]. BOD can be measured by various methods such as dilution method with use of the Winkler method [7], manometric

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method by Sierp and Warburg and automatic methods including those with the use of Sapromat [7], respirometric method [18] etc. One of latest methods is the use of the system Oxi Top Control which determines the use of oxygen through the measurement of pressure changes [11].

2. MODELLING OF BOD

The changes in the oxygen concentration in the surface water are described with the equation of Streeter and Phelps and they depend on saturation, reaeration and changes in the concentration of the contaminations described by BOD [1]. Adrian and Sanders showed that the modelling of time dependence of BOD can be described with the equations of the half order (the index of 0.5) [1], and of the first or the second order [2] with the results showing no significant differences.

The following equation is most often used to model the course of the BOD

\[
\frac{dy}{dt} = ky
\]  

(1)

which sometimes written as

\[
\frac{d(L - y)}{dt} = K(L - y)
\]  

(2)

or [6]

\[
\frac{dy}{dt} = K(L - y)
\]  

(3)

where \(y\) is the actual value of BOD at time \(t\), \(L\) – total BOD, whereas \(k, K\) – reaction rate constants. \(y(t)\) is an increasing function, whereas \((L - y)(t)\) – decreasing one which is shown in Fig. 1.

![Fig. 1. Tendency of changes in the dependence of \(y\) and \(L - y\) on time [10]](image-url)
After integrating the Eq. (3) Fujimoto [6] obtained

\[
\frac{L - y}{L - y_0} = e^{-kt} = 10^{-kt}
\]

where \(y_0\) is the BOD5 value at time \(t = 0\).

The plot of the rectilinear dependence of \(y(t + 1)\) on \(y(t)\) (Fig. 2) enabled one to determine the intersection point of the vertical axis and the parameters \(k\) and \(L\).

Thomas [19] put forward a function which allows one to receive the dependence while being developed in the series:

\[
\left( \frac{t}{y} \right)^{1/3} = \frac{1}{(2.3kL)^{1/3}} + \frac{(2.3k)^{1/3}}{6L^{1/3}} t
\]

which plotted as the dependence of \((t/y)^{1/3}\) on \(t\) becomes a linear function, which allows to define the intersection point \(A\) and its slope. Based on the values of \(A\) and \(B\) one may calculate \(k\) and \(L\):

\[
k = \frac{2.61B}{A}, \quad L = \frac{1}{2.3kA^3}
\]

Navone [16] suggested the following procedure based on Eq. (3). After comparing the time measurement \(t_i\) and corresponding values of BOD \(y_i\) \(\frac{dy_i}{dt}\) should be calculated from the equation

\[
\frac{dy_i}{dt} = \frac{y_{i+1} - y_{i-1}}{t_{i+1} - t_{i-1}}
\]
The plot of the dependence of \( \frac{dy_i}{dt_i} \) on \( y_i \) is linear; hence based on its slope (equal to reaction rate constant) and intersection point \( L, \frac{dy_i}{dt_i} = a \) can be calculated, which equals \( a/k \). As the author states, the method gives the results very similar to those obtained by Moore et al. [15].

Numerous authors raised the doubts concerning the application of the equation of the first order to describe BOD. Adrian and Sanders [1], analysing the equation of Streeter and Phelps, introduced the differential calculus of the half order of the form:

\[
\frac{y}{t^2} = k_{1/2} \cdot \frac{L^{1/2}}{t} - \frac{k_{1/2}^2}{4}
\]

(8)

From the linear dependence of \( y/t^2 \) on \( 1/t \), the constant \( k_{1/2} \) and \( L \) may be determined.

Marske and Polkowski [13] suggested the application of the model of the first order where \( k < 0.2 \) (Eq. (9)) and otherwise the model of the second order (Eq. (10))

\[
y = L \left( 1 - e^{-kt} \right)
\]

(9)

\[
y = \frac{t}{kL + \frac{1}{L}} t
\]

(10)

The doubts concerning the orders of the model were also raised by Young and Clark [22], Hewitt et al. [9], Borsuk and Stow [4] and Mason et al. [14].

Young and Clark [22] compared the results obtained for the model of the first and the second orders. For the reaction of the second order they obtained

\[
\frac{d(L - y)}{dt} = K (L - y)^2
\]

(11)

and after some simple modification

\[
y = \frac{t}{1 + \frac{1}{L}} = \frac{t}{a + bt}
\]

(12)

they arrived at simple linear equation

\[
\frac{t}{y} = a + bt
\]

(13)
Using the least squares method, first the parameters \( a \) and \( b \) can be calculated (Eqs. (14) and (15)) and then the values \( L \) and \( K \):

\[
a + b \left( \frac{\sum t}{t} \right) - \left( \frac{\sum y}{t} \right) = 0
\]

\[
a + b \left( \frac{\sum t^2}{\sum t} \right) - \left( \frac{\sum t^2}{\sum t} \right) = 0
\]

Hewitt et al. [9] analysed the model of many orders based on the equation:

\[
- \frac{dL_i}{dt} = K L_i^n
\]

which after integrating and transforming changes into the form:

\[
y = L - \left( (n - 1) K t + L^{(1-n)} \right)^{1/(1-n)}
\]

The authors calculated \( K \) and \( L \) for the equations of the orders from 1 to 4.0 with the step every 0.5. They proved that the higher the value of the order \( n \) is, the lower \( K \) and the higher \( L \) is. They also found that the values \( L \) obtained based on the equation of the second order are (1.23–1.55)-fold (1.39-fold on average) higher than those for the first order. Young and Clark [22] obtained similar values and with the ratio of about 1.25. As Hewitt et al. proved [9], the lowest average square error related to the experimental and calculated \( L \) was for the equations of the second order.

Borsuk and Stow [4] interpreted Eq. (16) in another way, assuming the value \( L_i \) being other BOD. Therefore, after integrating and substituting the relation between \( L_i \), \( y \) and \( L \) as \( L_i = L - y \), one obtains:

\[
y = L - \left( L^{1-n} - k_n t (1-n) \right)^{1/(1-n)}
\]

where \( n \) was called a pseudo order of the equation, \( k_n \) – reaction rate constant of the mixed order expressed as (mg⋅dm\(^{-3}\))\(^{(1-n)}\)⋅24 h\(^{-1}\).

Modelling the BOD changes in the sewages, Mason et al., [14] suggested the application of the double exponential model. The model suggested by them is expressed by the formula

\[
y = L_1 \left( 1 - e^{-k_1 t} \right) + L_2 \left( 1 - e^{-k_2 t} \right)
\]
where $L_1$ and $L_2$ are the total BOD relatively. The subscript 1 refers to the contamination decomposed very easily and very quickly, and subscript 2 to the contamination decomposing more difficult and slower. The substances marked with the subscript 1 are the contamination which can also be decomposed in anaerobic conditions or even, at the presence of oxygen, due to its fast consumption, they can produce high temporary deficit of oxygen and periodical occurrence of the anaerobic conditions may occur. The values $k_1$ and $k_2$ are the reaction constant rates. The total of $L_1$ and $L_2$ gives the total value of the total $L$. The double model is characterized by higher accuracy in comparison with the single model expressed by the formula

$$\frac{Y}{L} = 1 - 10^{-kt}$$

The average square error for the double model was about 70% lower than such an error for the single model.

Anthonisen et al. [3] and Dmitruk and Dojlido [5] examined the influence of the nitrification on the measured value of BOD$\textsubscript{5}$.

3. MATERIALS AND METHODS

The sewage samples were taken from the sewage treatment plant in Piaseczno. The raw sewage was taken from the technological process after the crates and the grid chamber. These sewages were characterized by high concentration of organic compounds (BOD) ranging from 400 to 1500 mgO$\textsubscript{2}$·dm$^{-3}$. The research was done from April to January in order to take into account changes resulting from the natural behaviour of dwellers. The sewages were examined in 18 measurement series, from 3 to 10 samples in each, which means the total of 92 samples.

The measurements were made at the most 2 h after collection. The value of BOD was measured in each sample with the use of the apparatus OxiTop Control produced by the WTW Company. Each sample was placed in a special bottle made of dark glass and closed with the head with the seal containing NaOH which played a role of an absorbent of CO$\textsubscript{2}$ created during the decomposition of organic compounds. The measurement of BOD was based on the decrease in the gas pressure in the bottle due to use of oxygen in decomposition of organic compounds and formation of CO$\textsubscript{2}$, which was absorbed by NaOH in the sea. Each bottle with the magnetic mixer was placed in a thermostatic chamber.

The head enabled the measurement of BOD at any time and enabled the measurement of the changes in BOD at the time of a given measurement step. The measurement step was set for 180 min; therefore during 5 days 40 points were recorded for each sample.
In order to examine the influence of the nitrification on BOD, two series of measurements were made. Each sample was placed in a measurement bottle containing only sewages, and to the other containing the same sewages N-allylthiourea (C₄H₈N₂S) as a nitrification inhibitor was added.

Five models described in this study were verified:

- Model of Thomas [19] described by Eq. (5) enabling determination of the reaction rate constant $k$ and the total BOD $L$ with the use of Eq. (6).
- Model of Navone [16] described by Eq. (7).
- Model of Fujimoto [6] described by Eq. (3).
- Model of Adrian and Sanders [1] enabling one to determine the reaction rate constant $k_{1/2}$ and the total BOD by means of Eq. (10).
- Models of Young and Clark [22]) and Adrian and Sanders [2], enabling determination of the reaction rate constant $k_2$ and the total BOD from Eq. (12).

4. RESULTS AND DISCUSSION

The sewage samples used in the research were characterised with quite a significant dispersion of the BOD₅ concentration (Fig. 3). The measurement points corresponding to samples without inhibitor of nitrification are marked with squares those containing the inhibitor – with asterisks.

![Fig. 3. BOD₅ concentrations in the analysed samples](image)

The inhibitor influenced the value of BOD₅ as the points marked with the squares correspond to higher values of BOD₅ than those marked with asterisks. However, it is problematic to find any correlation based on this diagram as the distance between the points in each pair of the results differs a lot and it seems that it does not depend on the BOD₅ value. It can be noticed that in some cases the asterisks are located higher than the squares – it should not be treated as a regularity but rather as a measurement
error difficult to explain. Rejecting the results in which BOD$_5$ with the inhibitor took the values higher than without the inhibitor, the average value of the percentage deviations calculated from Eq. (20) was calculated.

$$A = \frac{(\text{BOD}_5 \text{ (without)}) - (\text{BOD}_5 \text{ (with)})}{(\text{BOD}_5 \text{ (without)})}.100\%$$  \hspace{1cm} (20)

The average value of the deviations calculated for each pair of points equalled 16.6%. However, the calculated standard deviation which equalled 14.1% confirmed of the lack of reasonable correlation of the influence of the inhibitor on the decrease in the BOD$_5$ value.

5. MODELS

Almost in all the measurement series the model by Thomas gave quite good compatibility and the measurement points apart from the initial area of the adjustment were located along a coherent straight line (Fig. 4).

Fig. 4. Approximation of the measurement points of an exemplary series according to the model by Thomas

The functions which were obtained by the approximation of the measurement points were marked with rectangles in Fig. 3. The effect of the approximation is excellent because the values of $R^2$ are very high, 0.993 and 0.9743, respectively. The points marked with asterisks refer to the sample without the inhibitor, the samples marked with circles refer to the sample with the inhibitor, where the process of nitrification was stopped. As is seen in the model, it can be easy to define the lag phase as it is marked with the points lying outside the line. In this measurement series, the effect of adjustment finishes at the end of the first 24 h, and the process occurs according to the foreseen models in the next ones. Similarly substantial differences in the initial range
Analysis of chosen models describing the changes in BOD\textsubscript{5} in sewages 69

of time are visible in the models whose approximating functions with reference to the measurement points are presented in Fig. 5.

![Fig. 5. Approximation of the measurement points of an exemplary series according to the models: a) Navone, b) Fujimoto, c) Adrian and Sanders [1], d) Young and Clark, and Adrian and Sanders [2]](image)

The negative values in the model by Navone presented in Fig. 5 seem to be puzzling. Their occurrence does not result from the physical phenomena but from the measurement method. During the measurement by the electronic method with the small step of the reading (every 3 h) there were the cases when the next measurement had the same value as the previous one or even smaller. Because in this method the vertical axis constitutes of the increases between the previous point and the next one, when the curve is increasing, the sign of the function is positive and when it is decreasing, it is negative. Therefore, paradoxically the more precise the measurement is i.e. the more often BOD\textsubscript{i} readings with the hour step or lower, the more often such a situation can occur. Owing to this fact, the dispersions of the measurement points located in this diagram are immense and therefore the coefficient $R^2$ indicated very low values. If the 24 h readings are applied (5 measurements), then such a situation does not occur but the risk of making a mistake will increase with regard to the small number of the measurement points. It is worth comparing $R^2$ obtained for all the methods, taking into consideration that the same measurement series were used to demonstrate them.

Using the equations derived from the approximation of the measurement points, the reaction constant rate $k$ and the final BOD were calculated according to the equations presented in Table 1.
Equations to calculate the constant $k$ and the final value of BOD

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>Equation</th>
<th>BOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thomas [19]</td>
<td>$k = \frac{2.61 a}{b}$</td>
<td>$BOD = \frac{1}{(2.3kb^3)}$</td>
</tr>
<tr>
<td>2</td>
<td>Navone [16]</td>
<td>$k = \frac{b}{BOD}$</td>
<td>$BOD = \frac{b}{(-a)}$</td>
</tr>
<tr>
<td>3</td>
<td>Fujimoto [6]</td>
<td>$k = -\log a$</td>
<td>$BOD = \frac{b}{(1 - 10^{-k})}$</td>
</tr>
<tr>
<td>4</td>
<td>Adrian and Sanders [1]</td>
<td>$k(1/2) = (4b)^{0.5}$</td>
<td>$BOD = \frac{(a/k)^2}{b}$</td>
</tr>
<tr>
<td>5</td>
<td>Young and Clark [22]</td>
<td>$k(2) = \frac{1}{(BOD^2b)}$</td>
<td>$BOD = \frac{1}{a}$</td>
</tr>
</tbody>
</table>

$a$ is the slope of the linear equation, $b$ is the constant term.

To check the vulnerability of the final results ($k$ and BOD) to the way of their presenting, all the diagrams and calculations with reference to various time units were analysed. The calculations were made for the readings expressed in min, every 180 min = 3 h.

Then the analysis was made of the influence of the indirect measurement values, i.e. BOD$_1$, BOD$_2$, BOD$_3$, BOD$_4$ and BOD$_5$ on the reaction rate constant $k$ and the final calculated value of BOD marked as $L$. Dependences $k = f(BOD_1)$, $k = f(BOD_2)$, ..., $k = f(BOD_5)$ and $BOD = f(BOD_1)$, $BOD = f(BOD_2)$, ..., $BOD = f(BOD_5)$ were plotted for each method. The conclusion can be drawn that the distributions of the points on the plots with $x$-axis corresponding to BOD$_1$, BOD$_2$, ..., BOD$_5$ both with reference to $k$ and BOD were similar. Therefore, in Figs. 6 and 7 only the dependences $k = f(BOD_5)$ and $BOD = f(BOD_5)$ referred to each of the analysed methods.

As can be seen in Figs. 6 and 7, in the plots based on the methods by Thomas, Navone and Fujimoto no dependences of the reaction constant rates $k$ are visible. Applying the method by Adrian and Sanders [1] leads to linear function $k = f(BOD_5)$, and the plots $k = f(BOD_1)$, $k = f(BOD_2)$, ..., $k = f(BOD_5)$ are similar, differing only with the dispersion of the points. Obviously, the biggest dispersions of the points in all the functions were observed with reference to the $x$-axis expressed with BOD$_1$, the smaller ones with reference to BOD$_2$, etc. which resulted from the reading fluctuations in the initial stages of each research series. As regards the plot of the function of the second order presented in Fig. 7, one may only conclude that it is a decreasing function with marked linearity but dispersion of measurement points prevents accurate analysis.

The plots $BOD = f(BOD_5)$ are almost linear increasing functions similarly as functions of BOD$_1$, BOD$_2$, ..., BOD$_5$ differing only with the accuracy of approximation; for $f(BOD_1)$ $R^2$ oscillated around 0.6, for $f(BOD_2)$ around 0.7, for $f(BOD_3)$ around 0.8, but for $f(BOD_4)$ and $f(BOD_5)$ it reached 0.9.

Based on the above results, it is seen that the BOD measurements in the natural conditions, i.e. referred to the sewages flowing from the analysed places are characterized by high changeability and it is difficult to determine explicitly the amount of the consumed oxygen with respect to the decomposition of organic compounds.
Fig. 6. Dependences of $k$ on the final measurement values – BOD$_5$: a) Thomas, b) Navone, c) Fujimoto, d) Adrian and Sanders [1], e) Young and Clark

Fig. 7. Dependences of BOD on the final measurement values – BOD$_5$: a) Thomas, b) Navone, c) Fujimoto, d) Adrian and Sanders [1], e) Young and Clark
Therefore, numerous series of experiments were a good chance to make the comparative analysis verifying the models with regard to their accuracy of the description of the oxygen changes in the sewages. For each measurement series and each analysed model, the calculations of time dependences of BOD were made and compared to the experimental results. Plots of the functions together with the measurement points for exemplary series without the inhibitor and with the inhibitor are shown in Fig. 8.

![Exemplary plots of the analysed functions together with the measurement points for samples without (a) and with the nitrification inhibitor (b). Models: 1 – Navone, 2, 3 – Fujimoto, Young and Clark, 4 – Thomas, 5 – Adrian and Sanders](image)

The best matching was obtained with the use of the differential equation of the second order. A little worse results were obtained for the model by Fujimoto. The plots obtained according to the model by Thomas are located far below the measurement points while those based on the model by Navone correspond to much higher values. The plot of the differential equation of the half order shows a distinct maximum in the area between the third and fourth 24 hours. Therefore, in the areas above the fourth 24 hours the values of the function decrease, showing the lower and lower values discordantly to the essence of the occurrence of the oxygen decomposition of the organic compounds.

The maximum of the curve \( k(1/2) \) can be calculated after equating to zero the derivative of Eq. (8). One obtains \( t = 2L^{0.5}/k_{1/2} \). Having calculated for all the samples the values at the maxima, the average value \( t = 3.24 \) day was calculated with the standard deviation of 0.66 day for the series without the inhibitor, and for the series with the
inhibitor 3.04 day with the deviation of 0.52 day. For the samples with the inhibitors, the maximum is shifted to the left by 0.2 day, i.e. in the direction of the lower values. The function of the half order is located quite well in relation to the measurement points only to reach the maximum.

The calculations for all the measurement series presented the following: the average value \( t = 3.2 \) day for the series without the inhibitor with the standard deviation of 0.65 day, for the series with the inhibitor the average value \( t = 3.04 \) day with the standard deviation of 0.52 day. The median of 3.15 for the measurement series without the inhibitor and 2.94 for the series with the inhibitor were obtained.

The objective way to evaluate the applicability of individual functions with respect to the measurement points is the definition of the deviation \( \delta \)

\[
\delta = \left( \frac{1}{N} \sum_{i=1}^{N} \left( \frac{BOD(t)_{\text{meas}}}{BOD(t)_{\text{calc}}} \right)^2 \right)^{0.5}
\]

where \( BOD(t)_{\text{meas}} \) are the results of the BOD measurements at time \( t \), \( BOD(t)_{\text{calc}} \) are the BOD values calculated with the use of the individual models, and \( N \) is the number of the measurement points.

The arithmetic averages of the deviation values together with the standard deviations were presented in Table 2.

### Table 2

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>Without the inhibitor</th>
<th>With the inhibitor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>1</td>
<td>Thomas</td>
<td>222.5</td>
<td>81.2</td>
</tr>
<tr>
<td>2</td>
<td>Navone</td>
<td>195.8</td>
<td>94.5</td>
</tr>
<tr>
<td>3</td>
<td>Adrian and Sanders</td>
<td>135.1</td>
<td>83.4</td>
</tr>
<tr>
<td>4</td>
<td>Young and Clark</td>
<td>27.1</td>
<td>14.6</td>
</tr>
<tr>
<td>5</td>
<td>Fujimoto</td>
<td>48.1</td>
<td>31.8</td>
</tr>
</tbody>
</table>

As it can be observed, the most detailed description is obtained using the model of the second order, which is proved not only by the average of the derivations but also the low value of the standard deviation showing the regularity of the description. Also, the model by Fujimoto was accepted, but it shows the tendency to scatter the results which suggest the high, in comparison with the averages, standard deviations.

The influence of the nitrification inhibitor within 5 days was not very significant, but the lower values of BOD in the series with the addition of the inhibitor in comparison with the series without it could be noticed. Incidentally, in some series its influ-
ence was not noticed but in most of them it was distinct. To illustrate this dependence, functions of BOD with inhibitor = \( f(\text{BOD without inhibitor}) \) were plotted (Fig. 9). For each sample, almost linear plots were obtained which enabled to approximate the measurement points with the linear function. The slope of the straight line was a measure of the inhibitor influence. Its value lower than 1 indicated inhibition of nitrification. In some cases, the slope was higher than 1.0 which proved the error and such series were rejected. As is seen in Fig. 9, the influence of the inhibitor was distinct in this series.

![Fig. 9. Dependence of BOD after using the inhibitor of nitrification on the value of BOD with no inhibitor in an exemplary sample](image)

After analysis of all measurement series, the following statistical results were obtained:

- the average value of the slope – 0.895,
- the standard error – 0.0114,
- the median – 0.915,
- the standard deviation – 0.1062,
- the variance – 0.0112.

The nitrification takes place in the period of the first 5 days of incubation because the nitrification inhibitor significantly decreased the BOD values. Approximately, it can be said that the nitrification includes about 10% of the BOD\(_5\) value.

6. CONCLUSIONS

Numerous factors influence the distribution of points defining the changes in BOD, which include the concentration of the organic contamination and various disturbing factors such as salinity, poisons, etc. Kovarowa-Kovar and Egli [12] found
that the growth of microorganisms is limited both by the availability of organic carbon and its limitations as well as it depends on the access to other nutrients such as nitrogen and phosphorus. The following conclusions and insights can be based on the presented results and analyses of the models.

- It is very easy to define the time of the adjustment of microorganisms with the use of the model by Thomas. The measurement points in the first hours of the first day distinctly deviate from those of the next hours (Fig. 4). Analysing all the measurement series, it could be noticed that not in every case a minimum existed, sometimes the whole scope was positive, but then the slope of the straight line approximating the points in the initial hours was, as a rule, higher than in next ones.

- The model by Navone creates a certain paradox. The more measurement points, i.e. more often measurements are made, the lower the coefficient of determination $R^2$. Additionally, with regard to the fact that the increases in the BOD$_5$ values between the neighbouring points are marked on the y-axis, the signs of these changes can be both positive and negative. It causes an inevitable measurement error. Such a situation does not exist if the measurements are made in longer intervals, for example every 24 h or 12 h. In spite of low values of $R^2$ and scatter of the points in the plot based on the Navone model, the values $K$ and $L_0$ are determined reasonably.

- The model by Fujimoto is quite easy to use but the final equation modelling the the BOD changes creates a trap. Fujimoto [6] originally published the equation

$$y(t + h) = y(t) \times 10^{-kh} + \left(1 - 10^{-k \cdot h}\right)L$$

which, as it occurs, is not correct when the measurement of BOD is made with the step of 1 day. However, if the measurements are made more often (like in this case), for example every 3 h, the calculated values of BOD are lowered with respect to the measured ones. Therefore, regardless of the frequency of the measurement in the above formula, $h = 1$ should be inserted and then the results close to the measured values are obtained.

- Nitrification occurs in the first 5 days of incubation as the nitrification inhibitor distinctly lowered the measured values. It can be said that the nitrification includes approximately about 10% of the value of BOD$_5$.

SYMBOLS

$k$ – first order reaction rate constant
$K$ – higher orders reaction rate constant
$n$ – reaction order
$t$ – time, days or h
$y$ – BOD concentration at time $t$, mg O$_2$·dm$^{-3}$
$y_0$ – initial BOD concentration, mg O$_2$·dm$^{-3}$
$k_1, k_2$ – reaction rate constant in the model of Mason
\( L_1 \) – total BOD referring to contamination which decomposes very easily and quickly
\( L_2 \) – total BOD referring to contamination which decomposes more difficult and slower
\( L \) – total BOD, mg O\(_2\cdot dm^{-3}\)

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