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COEFFICIENTS OF LONGITUDINAL AND TRANSVERSAL DISPERSION OF MASS ON THE GRATES FOR WASTE INCINERATION SYSTEMS

The paper describes the methods of evaluating longitudinal and transversal dispersion coefficients for forward moving and reciprocating grates. The methods are based on both theoretical and dimensional analyses. The values of the coefficients obtained theoretically at various parameters of material (density, particle diameter) and various parameters of grate movement (grate bars velocity, inclination angle) were verified using two methods. The first method is based on the analysis of the Peclet number calculated from the residence time distribution. The second one is based on the comparison of the concentration field determined experimentally with the values obtained by mathematical method. Both methods give satisfactory results.

1. INTRODUCTION

Grate is a very important element in the waste incineration system. The grate should allow the transport, mixing and air distribution which ensures a complete combustion of waste. Two types of grate are commonly used: forward moving grate and reciprocating grate.

In the paper [1], the theoretical formulas for determining the parameters important for the mass transport were defined and the methods of their testing were proposed. These parameters are as follows: residence time distribution, dispersion coefficients and the mixing rate of material. The mechanism of dispersive transport of the material on the mixing grate was described by longitudinal and transversal dispersion coefficients. The dispersion coefficient D_L represents the mass transport along the main grate axis, while the coefficient D_p represents the transversal transport.

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An analogy between the combustion chamber with moving grate and chemical flow reactor was assumed [2] and mathematical formulas for real residence time of material in the reactors of these types were applied.

2. THEORETICAL ANALYSIS OF DISPERSION COEFFICIENTS – DIMENSIONAL ANALYSIS

The dimensional analysis was applied to define the formulas for the longitudinal and transversal dispersion coefficients [3].

2.1. DIMENSIONAL ANALYSIS OF LONGITUDINAL DISPERSION COEFFICIENT

It was assumed that the coefficient D_L was related to the following parameters: grate velocity (m/s), particle diameter (m), bulk density (kg/m^3), and mass flow rate (kg/s):

$$D_L = f(u_R, d_p, \rho_n, \dot{m}) . \tag{1}$$

For monodispersive layer with ρ_n = const, d = const, this formula can be rewritten as follows:

$$D_L = C \cdot \frac{\dot{m}}{d_p \cdot \rho_n} , \tag{2}$$

while for polydispersive layer it assumes the following form:

$$D_L = C \cdot \dot{m} \cdot \sum_i \frac{1}{d_{pi} \cdot \rho_{ni}} , \tag{3}$$

where:

D_L - longitudinal dispersion coefficient (m^2/s),

C – coefficient dependent on the type of grate (forward moving grate, reciprocating grate).

2.2. DIMENSIONAL ANALYSIS OF TRANSVERSAL DISPERSION COEFFICENT

It was assumed that D_p depends on the following parameters:

$$D_p = f(u_R, d_p, \varepsilon) . \tag{4}$$

In the dimensional analysis, the porosity of the layer ε was not taken into account because of its non-dimensionality, and it was added to the final forms of formulas.

For monodispersive layer:

$$D_p = C \cdot \frac{u_R \cdot d_p}{\varepsilon}, \tag{5}$$

for polydispersive layer:

$$D_p = C \cdot u_R \sum_i \frac{d_{pi}}{\varepsilon_{pi}}, \tag{6}$$

where:

D_p – transversal dispersion coefficient (m^2/s),
C – coefficient dependent on the type of grate.

3. VERIFICATION OF DISPERSION COEFFICIENTS

Two methods were applied to verify the dispersion coefficients determined by mathematical methods. The results obtained are the values of the constant C in equations.

3.1. THE FIRST METHOD OF VERIFICATION

Peclet number and real residence time distribution (RTD) of particles on grate

A real residence time of particles on the grate is a very important parameter in the calculations representing the combustion processes. In batch reactors, the residence time is the same for each particle and easy to determine. The same holds for the plug-type reactors [4]. In real flow reactors (as in combustion chamber with moving grate), the residence time is a stochastic function and is not easy to determine.

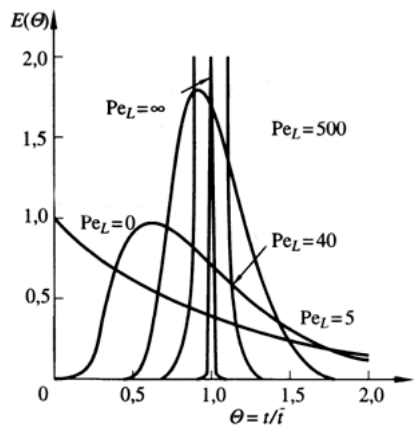

Fig. 1. $E(t)$ distribution as the function of Peclet number [4]

Theoretical and real dispersion coefficients are compared based on the results of investigting RTD on the experimental stand and the results of their processing using computer program [5]. The $E(t)$ distributions and the functions of the Peclet number are shown in figure 1.

The difference in the residence time of particles on the grate related to an average time is called variation. For the stimulus coming from an impulse, the variation is commonly expressed by:

$$\sigma_i^2 = \frac{\Sigma t_i^2 E(t_i)\Delta t_i}{\Sigma E(t_i)\Delta t_i} + t_m^2 , \tag{7}$$

where:

t_i – the discharge time of a marker in input–output scheme (s),

t_m – average residence time of material on the grate (s),

Δt_i – the range of the discharge times of marker samples.

Bearing in mind that for open system relation (8) is valid:

$$\frac{\sigma_t^2}{t_m^2} = \frac{2}{Pe_L^2}(Pe_L + 4) , \tag{8}$$

one can determine the value of diffusive Peclet number, being the measure of mixing intensity. This value of Pe allows the classification of the mass flow into the flow of intensive or poor mixing, hence it has a decisive influence on the correctness of calculations. Namely, if the condition $Pe_L - 1 > 0.01$ is not fulfilled (poor mixing intensity in the reactor) the average residence time of the material in reactor (on the grate) according to LEVENSPIEL [4] can be calculated as follows:

$$t_m = \frac{\Sigma t_i c_i \Delta t_i}{\Sigma c_i \Delta t_i} , \tag{9}$$

where c_i is the mass concentration of tracer (kgi/kg).

The dispersion coefficient was calculated from the relation for the Pe number. The Pe can be calculated from relation (8), and in consequence from the basic relation $Pe_L = u_R L/D_L$ the longitudinal dispersion coefficient can be determined:

$$D_L = \frac{u_R L_R}{Pe_L} , \tag{10}$$

where:

L_R – the length of the grate (m),

u_R – an average velocity of material (waste) on the grate (m/s),

Pe_L – the diffusion Peclet number.

The diffusion Peclet number describes mixing intensity and determines the type of the flow.

The computer code calculates the residence time distribution (RTD) of the material on the grate and the value of the diffusion Peclet number.

The other parameters, i.e. L_R and u_R, are determined either by measurement or by calculations of RTD.

From the analysis of the values obtained for a forward moving grate with wooden balls, keramsite balls, biomass particles, wood and their mixtures the approximate values of dispersion coefficients were determined. Nevertheless, some inconsistency was observed, especially for the mixtures of materials. Also significant inconsistency was observed at low velocities of grate bars (u < 3 mm/s).

3.2. THE SECOND METHOD OF VERIFICATION

Mass concentration field determined by the Mathematica code [6]

In the other method of the verification of dispersion coefficients, their calculated values were inserted into the differential equations describing concentration fields. The following equations were solved by the Mathematica code:

a) Equation with longitudinal dispersion coefficient. For the equation representing the forward moving grate only longitudinal dispersion coefficient was assumed, because, as experiments show, transversal dispersion can be neglected. The mass concentration field in one-dimensional system as the function of grate length and time can be described by the equation:

$$\frac{\partial c(x,t)}{\partial t} = D_L \frac{\partial^2 c(x,t)}{\partial x^2} - u(x,t)\frac{\partial c(x,t)}{\partial x}. \qquad (11)$$

In the Mathematica code, this equation assumes the following form:

$$row = \partial_t c[x,t] == dl \cdot \partial_{x,x} c[x,t] - u[x,t] \cdot \partial_x c[x,t]. \qquad (12)$$

b) Equation with longitudinal and transversal dispersion coefficients. In reciprocating grate, the equation with two dispersion coefficients was applied (figure 2), because both coefficients influence the mass transport:

$$\frac{\partial c}{\partial t} = D_L \frac{\partial^2 c}{\partial x^2} + D_p \frac{\partial^2 c}{\partial y^2}. \qquad (13)$$

In the Mathematica code, this equation has the following form:

$$row = \partial_t c[x,y,t] == dl \cdot \partial_{x,x} c[x,y,t] + dp \cdot \partial_{y,y} c[x,y,t]. \qquad (14)$$

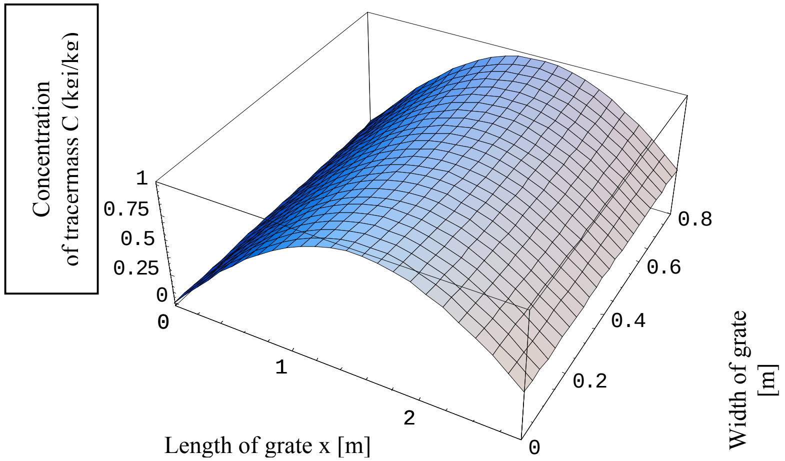

Fig. 2. Diagram showing the marker concentration on the grate
with both longitudinal and transversal dispersions [7]

The comparison of mass concentrations calculated from equations (11) and (13) using the coefficients obtained from equations (2), (3), (5) and (6) with the marker concentrations measured shows that better conformity of results was achieved for reciprocating grates (the discrepancy of the order of ≤ 20%). The greatest discrepancies between the calculated and measured concentrations arise for the outer bars of the grate for all time steps.

4. SUMMARY

The analysis of the calculated values of dispersion coefficients shown in figures 3 and 4 allows the following conclusions to be drawn:

- The values of longitudinal dispersion coefficient D_L are greater than those of transversal one, which proves that longitudinal direction of mass movement on the grate prevails.
- In the case of monodispersion materials, both longitudinal dispersion and transversal dispersion increase with an increase in the velocity of grate bars.
- The results of the measurements for polydispersive layer with a constant bulk density reveal the maximum of dispersion coefficients at a given velocity of grate bars. In the mixture of materials of various bulk densities, the values of both coefficients increase slowly with the velocity of grate bars.

The calculated and verified dispersion coefficients can be used for the modelling of mass and heat transport in the layers of fuel and waste on the grates of incineration systems. The results show that the method of solving differential equations for given initial and boundary conditions and for forward moving and reciprocating grates should be improved.

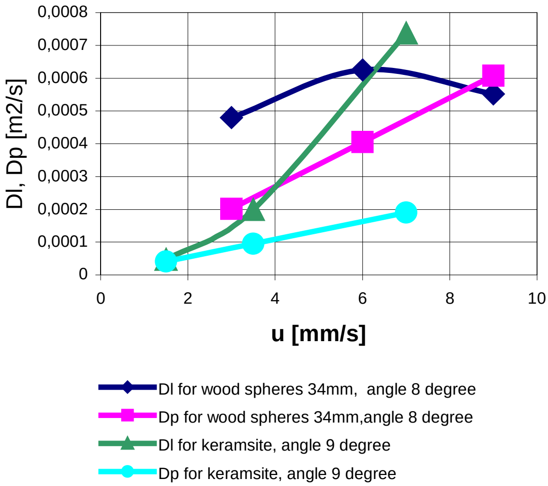

Fig. 3. Dispersion coefficients D_L, D_p versus the grate velocity for monodispersive layer

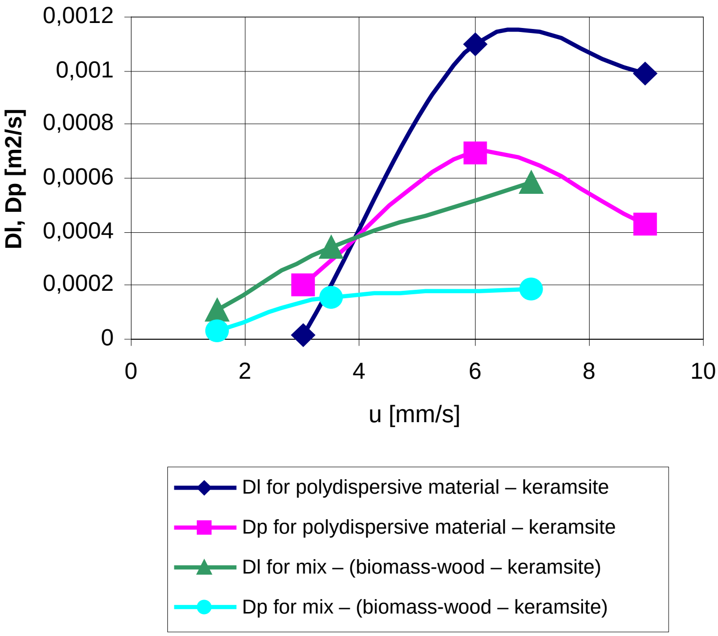

Fig. 4. Dispersion coefficients D_L, D_p versus the grate velocity for polydispersive layer

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